UC COURSE PROGRAM MODEL STRUCTURE AND CONTENT

IDENTIFICATION

COURSE	:	Introduction to Spectral Theory in Quantum Mechanics
TRANSLATION	:	Introducción a la teoría espectral en Mecánica cuántica
NUMBER	:	FIM3435
CREDITS	:	15 UC / 9 SCT
MODULES	:	2 theoretical modules
REQUISITES	:	FIZ0223, FIZ0313, FIZ0322
RESTRICTIONS	:	030401, 030501
CONECTOR	:	AND
CHARACTER	:	Optative
FORMAT	:	Theoretical modules
QUALIFICATION	:	Standard
KEY WORD	:	Spectral Theory, Quantum Mechanics y Functional analysis
FORMATIVE LEVEL	:	Master, PhD

I. COURSE DESCRIPTION

This is an introductory course to spectral theory and preparatory for physical-mathematical research in quantum mechanics. Topics of functional analysis, Lebesgue integration theory and operator theory will be studied. In this way, it is expected that students acquire the skills to understand, explain, and discriminate modern literature in Physics-Mathematics, being able to apply it to problems in quantum mechanics.

II. LEARNING OUTCOMES

- Interpret and effectively apply relevant mathematical results in quantum physics.
- Fluently master the mathematical language necessary to assess contemporary physical-mathematical literature.
- Analyze from the mathematical point of view, physical problems in quantum mechanics.
- Acquire the necessary knowledge to appreciate and prove relevant theorems in quantum physics.
- Develop communication skills for presentations and discussions in Physics.

III. CONTENT

1. Introduction
   1.1. Review of notions of Quantum Mechanics
   1.2. Review fundamentals of real analysis
   1.3. Sesquilinear forms and Schwarz's inequality

2. Hilbert and Banach spaces
   2.1. Scalar Products and Standards
   2.2. Notions of topology in normed spaces
   2.3. Orthogonality in Hilbert spaces

3. Linear operators in Hilbert spaces
   3.1. Bounded operators
   3.2. Unit projections and operators
   3.3. Single extension theorem

4. Integration of Lebesgue
   4.1. Intuitive construction of the Lebesgue integral
   4.2. Measure properties
   4.3. Convergence theorems

5. Fourier transform and Sobolev spaces
   5.1. Fourier transform
   5.2. Weak derivatives
   5.3. Sobolev spaces and their properties

6. Self-attached operators
   6.1. Basic criteria
   6.2. Kato-Rellich theorem
   6.3. Friedriechs theorem

7. Spectrum of a closed operator
   7.1. The resolvent and the spectrum
   7.2. Spectrum separation
   7.3. Weyl sequences

8. Spectral theorem and its applications in Quantum Mechanics
   8.1. The spectral measure
   8.2. Existence of solutions in the Schrödinger equation
   8.3. Analytical disturbance theory

IV. METHODOLOGICAL STRATEGIES

- Theoretical lectures, group projects, seminars.

V. EVALUATIVE STRATEGIES

- Weekly Homework: 60%
- Controls: 15%
- Final talk: 25%

VI. BIBLIOGRAPHY

Required:

- G. Teschl. Mathematical methods in quantum mechanics. American Mathematical Society, Providence, Rhode Island, 1999.
- J. Weidmann. Linear operators in Hilbert spaces, volume 68. Springer Science & Business Media, 2012.
- M. Reed and B. Simon. Methods of modern mathematical physics. I. Functional analysis. Academic Press, New York, 1972.

Optional:

- C. R. de Oliveira. Intermediate spectral theory and quantum dynamics, volume 54 of Progress in Mathematical Physics. Birkha user Verlag, Basel, 2009.
- Lawrence C. Evans. Partial differential equations, volume 19 of Graduate Studies in Mathematics. American Mathematical Society, Providence, RI, second edition, 2010.
- M. Reed and B. Simon. Methods of modern mathematical physics. IV. Analysis of operators. Academic Press, New York, 1978.
- Serge Richard. Operator theory on Hilbert spaces. [Online]. http://www.math.nagoya-u.ac.jp/~richard/teaching/s2019/Operators.pdf