



INSTITUTO DE FÍSICA
FACULTAD DE FÍSICA

COURSE	: MATHEMATICAL STRUCTURES IN QUANTUM MECHANICS
TRANSLATION	: ESTRUCTURAS MATEMÁTICAS EN MECÁNICA CUÁNTICA
NUMBER	: FIM3434
CREDITS	: 15 UC / 9 SCT
MODULES	: 2
REQUISITES	: (MAT1203 AND FIZ0313) OR (MAT1226 AND MAT2505)
RESTRICTIONS	: 030401, 030501, 020601, 020701
CHARACTER	: OPTATIVE
FORMAT	: THEORETICAL LECTURES
QUALIFICATION	: STANDARD
FORMATIVE LEVEL	: DOCTORATE
DISCIPLINE	: PHYSICS AND MATHEMATICS

I. COURSE DESCRIPTION

In this course we will present quantum mechanics in its modern formulation based on operator algebras (algebraic formulation). The algebraic formulation is a powerful tool that in recent decades has enabled incredible progress in understanding the fundamental ideas of quantum mechanics, statistical mechanics, and field theory.

This course aims to provide students of theoretical physics (Faculty of Physics) and mathematical physics (Faculty of Mathematics) with the basic notions of this theory through the description of the most important results obtained in this area. This is an "experimental" course that is designed to promote and strengthen interdisciplinarity between the Physics and Mathematics departments.

The course is essentially divided into three parts:

- a) Translation of physical concepts in a mathematical theory and axiomatization of basic structures.
- b) Application of the formalism to the quantum particle and connection with the standard formulation of Schrödinger's Quantum Mechanics.
- c) Application of formalism to quantum systems of many particles and statistical mechanics.

II. LEARNING OUTCOMES

Understand the fundamental mathematical structures of Quantum Mechanics and introduce important concepts and ideas around the duality of quantum physics - classical physics.

Introduce the basic concepts of the theory of Algebras of operators as a necessary tool for a rigorous investigation of various aspects of Quantum Mechanics.

III. CONTENT



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The central contents of the course are:

1. Mathematical description of a physical system:

- General description of a physical system.
- Comparison between classical mechanics and quantum mechanics.
- C^* -algebras, states and representations.
- von Neumann algebras.

2. Mathematical description of a quantum system:

- Heisenberg's indeterminacy relationship and non-commutativity.
- GNS construction and Gelfand-Naimark theorem.
- Probabilistic interpretation of quantum mechanics.
- Bell's inequalities- Quantum logic.

3. Kinematics and dynamics of the quantum particle:

- Weyl algebra and Heisenberg group.
- Von Neumann's uniqueness theorem and Schrödinger representation.
- Operators not bounded.
- Quantum dynamics.
- Quantum particle in a potential and scattering theory.
- Spin, Pauli exclusion principle and identical particles.

4. Quantum Mechanics of Large Systems and Statistical Mechanics:

- Balance and irreversibility.
- Infinite number of particles and Fock space.
- Quasi-local algebras of observables.
- Ising model.
- Statistical ensembles.
- Entropy.

IV. METHODOLOGICAL STRATEGIES

Theoretical lectures



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Homework
Seminars.

V. EVALUATIVE STRATEGIES

Homework and practical work (60%)
Exhibitions (40%)

VI. BIBLIOGRAPHY

REQUIRED:

- 1.F. Strocchi. *Mathematical Structure of Quantum Mechanics*, World Scientific, 2005.
- 2.B. C. Hall. *Quantum Theory for Mathematicians*, GTM 267, Springer, 2013.
- 3.G. Teschl. *Mathematical Methods in Quantum Mechanics: With Applications to Schrodinger Operators*, GSM 157, AMS, 2014R. Haag, *Local Quantum Physics*, Springer, 1992.

COMPLEMENTARY:

- 1.W. Thirring. *Quantum Mathematical Physics*, Springer, 2002.
- 2.M. Reed, B. Simon. *Functional Analysis (Methods of Modern Mathematical Physics vol. I)*, Academic Press, 1980.
- 3.N. Dunford, J. T. Schwartz. *Linear Operators Part 1: General Theory*, John Wiley & Sons, 1988.
- 4.W. Rudin. *Functional Analysis*, McGraw-Hill, 1991.
- 5.O. Bratteli, D. W. Robinson. *Operator Algebras and Quantum Statistical Mechanics vol. I & II*, Springer, 1997.
- 6.R. Haag, *Local Quantum Physics*, Springer, 1992.
- 7.D. Ruelle, *Statistical Mechanics: Rigorous Results*, World Scientific, 1999.
- 8.J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge U.P., 1987.

OPTIONAL:

N/A