

## Determining the efficiency of the hot-air engine as a refrigerator

### Objects of the experiments

- Measuring the heat  $Q_2$  extracted from the cylinder head during one revolution.
- Measuring the heat  $Q_1$  transferred to the cooling water during one revolution.
- Determining the efficiency  $\eta$  of the refrigerator.

### Principles

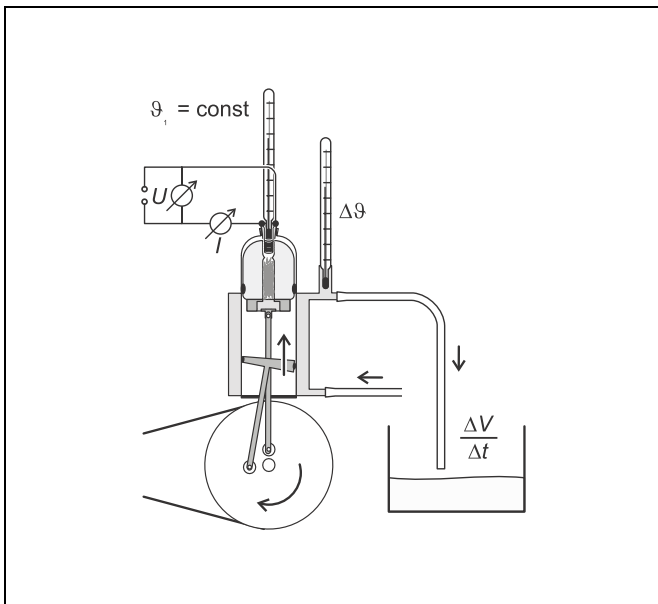
During one revolution, the hot-air engine, operated as a refrigerator, extracts the heat  $Q_2$  from the cylinder head and transfers the heat  $Q_1$  to the cooling water. As heat is transferred from the colder to the warmer reservoir, that is, in the opposite direction of spontaneous heat transfer, a certain amount of mechanical work  $W$  has to be supplied during one revolution. The refrigerator thus is a heat engine operated in the reverse direction. If there are no losses, the following relation between the quantities involved holds:

$$Q_1 = Q_2 + W \quad (I).$$

The efficiency of a refrigerator is defined as the ratio

$$\eta = \frac{Q_2}{W} \quad (II)$$

Determining the efficiency of the refrigerator



The efficiency is determined experimentally by driving the flywheel of the hot-air engine with an electric motor at an angular speed  $f$  and determining the electric calorific power that permanently keeps the cylinder head at room temperature in a compensation measurement. The electric work supplied per revolution is equal to the heat  $Q_2$  extracted from the cylinder head, that is

$$Q_2 = \frac{U \cdot I}{f} \quad (III)$$

$U$ : heating voltage,  $I$ : heating current

In addition, the increase in temperature  $\Delta\theta$  of the cooling water is measured and the power transferred to the cooling water

$$P = c \cdot \rho \cdot \frac{\Delta V}{\Delta t} \cdot \Delta\theta \quad (IV)$$

$c = 4.185 \text{ J g}^{-1} \text{ K}^{-1}$ : specific heat capacity of water,  
 $\rho = 1 \text{ g cm}^{-3}$ : density of water

$\frac{\Delta V}{\Delta t}$ : volume flow rate of the cooling water

is determined. From this we obtain the heat  $Q_1$  transferred to the cooling water during one revolution:

$$Q_1 = \frac{P}{f} \quad (V).$$

$f$ : rotational speed of the hot-air engine

The difference  $W' = Q_1 - Q_2$  of the heat quantities determined in the above-mentioned way is the mechanical work to be supplied during one revolution. It also contains the mechanical work  $W_R$  which is required to overcome the friction of the piston and causes additional warming of the cooling water (see experiment P2.6.2.1). The mechanical work to be supplied for the thermodynamic cycle, that is for the transfer of heat from the colder to the warmer reservoir, therefore is

$$W = Q_1 - Q_2 - W_R \quad (VI).$$

**Apparatus**

1 hot-air engine . . . . .	388 182
1 accessories for hot-air engine . . . . .	388 221
1 experiment motor . . . . .	347 35
1 control unit for experiment motor . . . . .	347 36
1 variable extra-low-voltage transformer S . . . . .	521 35
1 multimeter METRMax 2 . . . . .	531 100
1 multimeter METRMax 3 . . . . .	531 712
1 counter P . . . . .	575 45
1 slot sensor, infra-red . . . . .	337 46
1 transformer, 6 V~, 12 V~/30 VA . . . . .	562 73
1 adapter cable, 4-pole, 1.5 m long . . . . .	501 18
1 thermometer, $-10^{\circ}$ to $+40^{\circ}\text{C}$ . . . . .	382 36
1 plastic beaker, 1000 ml . . . . .	590 06
1 stopclock II, 60s/30 min . . . . .	313 17
1 stand base, V-shape, 20 cm . . . . .	300 02
2 stand rods, 25 cm . . . . .	300 41
connection leads (partly with $2.5\text{ mm}^2$ cross section)	
<i>additionally required:</i>	
open water vessel (at least 10 l)	
1 submersible pump 12 V . . . . .	388 181
1 low-voltage power supply . . . . .	522 16
2 silicone tubings, int. dia. $7 \times 1.5\text{ mm}$ , 1 m . . . . .	667 194
or	
cooling water feed and runoff	

**Setup**

The experimental setup is illustrated in Fig. 1.

**Temperature measurement in the cooling water:**

- Remove the GL14 screwing from the cooling-water outlet of the cylinder head, and mount the temperature adapter (c) from the accessories for hot-air engine (see instruction sheet 388 221).
- Insert the thermometer,  $-10^{\circ}$  to  $+40^{\circ}\text{C}$ , in the temperature adapter, and clamp it with the GL 18 screwing.

**Safety notes**

The glass components of the hot-air engine must not be exposed to excess thermal load.

- Mind the instruction sheet of the hot-air engine.
- Do not operate the hot-air engine without cooling water, and check whether the cooling-water circulation is flawless.
- Do not allow the temperature of the cooling water to exceed  $30^{\circ}\text{C}$  when the water enters the cooling circuit.

**Cooling-water supply:**

- Fill at least 10 l of water into the open water vessel, and hang the submersible pump in.
- Connect the output of the submersible pump to the cooling-water inflow of the hot-air engine, and guide the cooling water drain into the water vessel.
- Connect the submersible pump to the low-voltage power supply.

or

- Connect the cooling-water inflow of the hot-air engine to the tap, and guide the cooling-water drain to the runoff.

**Mounting the “thermometer with heater”:**

- Remove the filament of the “thermometer with heater” (d) (from 388 221) from the contact pins (see instruction sheet 388 221).
- Unscrew the screw gasket from the “cylinder-head cap with screw gasket” and screw the “thermometer with heater” on the cylinder-head cap.
- Put the filament back on the contact pins, and see to it that the filament does not touch the thermometer glass.
- Cautiously mount the cylinder-head cap on the cylinder of the hot-air engine.
- Turn the flywheel of the hot-air engine, and make sure that the filament does not touch the displacement piston in any position of the piston.
- While turning the flywheel check the packing of the hot-air engine; if necessary, close the hose nozzle for the pressure sensor with a stopper.
- Connect the variable extra-low-voltage transformer S to the thermometer with heater together with a voltmeter and an ammeter (measuring range 10 A).

**Drive:**

- Mount the electric motor, and connect it to the control unit.
- Put the driving belt around the flywheel and the driving disc, and tighten it by slewing the electric motor.

**Frequency measurement:**

- Attach the disc with holes from the accessories for hot-air engine to the crankshaft.
- Mount the slot sensor on the stand material, and align it with a hole of the disc with holes, the disc being at rest.
- With the 4-pole adapter cable connect the slot sensor to the 6-V output of the transformer (power supply, black plugs) and to the start input of the counter P (frequency measurement, red and grey plug).
- Drag the start input to the stop input, set the switch to “f”, and switch the counter P on.

**Measuring the cooling-water throughput:**

- Have the plastic beaker and the stopclock ready.

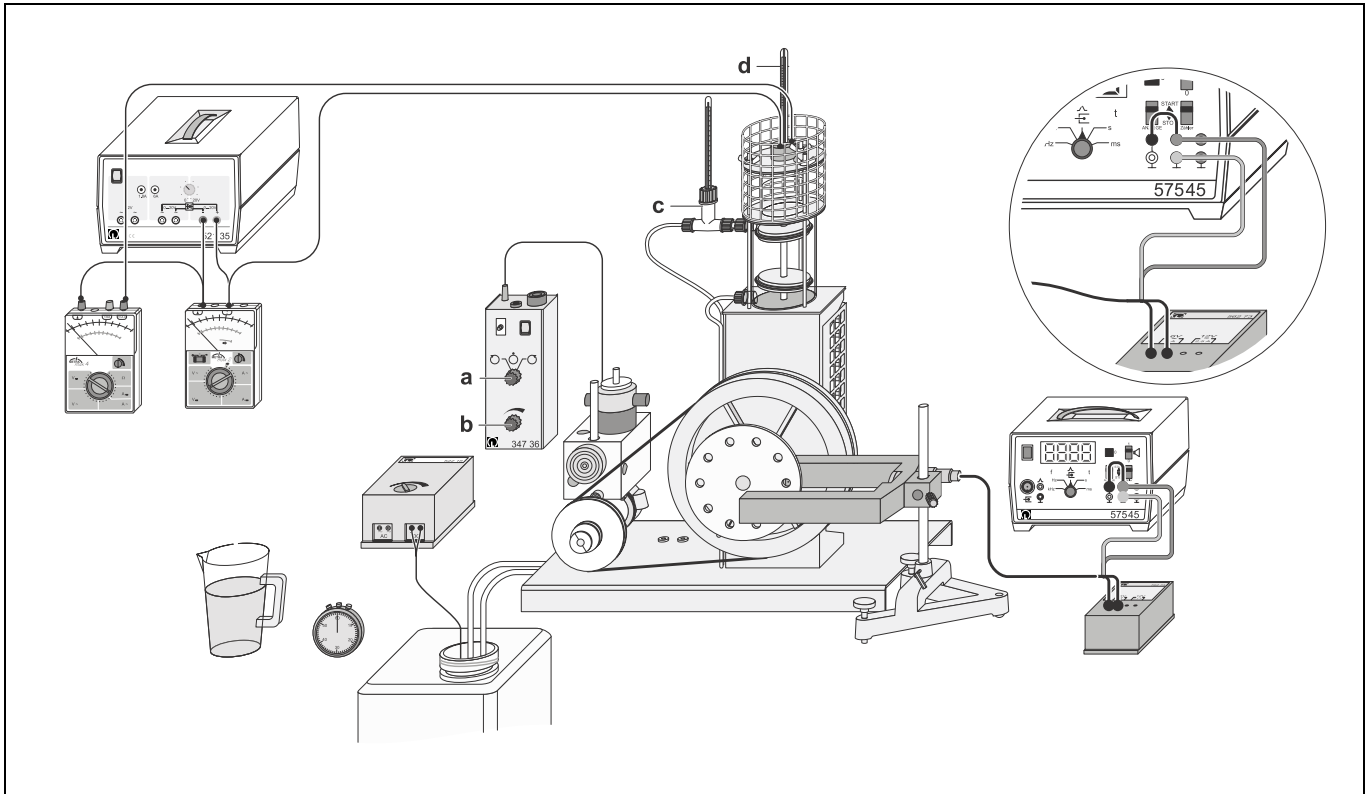


Fig. 1 Experimental setup for determining the efficiency of the hot-air engine operated as a refrigerator.

**Carrying out the experiment**

first:

- Switch the cooling-water supply on (for this, set, e.g., the low-voltage power supply to position 2), check the circulation, and wait until the water runs back through the outlet tubing.
- Put the end of the outlet tubing into the plastic beaker, and determine the volume throughput  $\Delta V$  of the cooling water per time interval  $\Delta t$  (see Fig. 2).
- Measure the temperature  $\vartheta_1$  in the cylinder head.
- Set the direction switch (a) to the middle position (standstill), set the speed adjusting knob (b) to the middle position, and switch the control unit on.
- Measure the temperature  $\vartheta$  of the cooling water every 2 minutes, and wait until the development of the temperature can be uniquely extrapolated.

then:

- Set the direction switch to clockwise running of the hot-air engine, and measure the rotational speed of the hot-air engine.

The rotational speed of the engine is obtained from the measured frequency and the number of holes in the disc with holes.

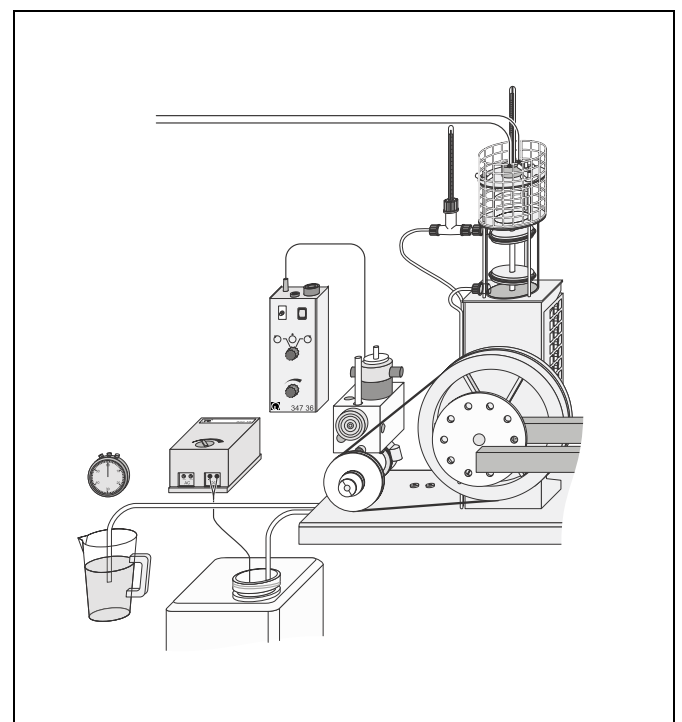
At the same time:

- Switch the variable extra-low-voltage transformer S on, and choose the heating voltage  $U$  so that the temperature  $\vartheta_1$  remains constant despite the operation of the hot-air engine as a refrigerator.
- If necessary, readjust the heating voltage  $U$ , and take it down together with the heating current  $I$ .

and:

- Continue measuring the temperature  $\vartheta$  of the cooling water every 2 minutes, observe the increase in temperature, and wait until the temperature has reached its maximum value.

Fig. 2 Determining the volume throughput  $\Delta V$  per time unit  $\Delta t$



next:

- Stop the clockwise motion of the motor, and continue observing the temperature of the cooling water.
- Determine the change in temperature  $\Delta\vartheta$  of the cooling water, and take it down.

Remark:

If the volume of the available cooling water is too small, the temperature of the vessel will also rise. The measured temperature change  $\Delta\vartheta$  has then to be corrected correspondingly.

- Vary the rotational speed with the speed adjusting knob (**b**), and repeat the measurement.

### Measuring example

Volume throughput of the cooling water: 780 cm<sup>3</sup> in 5 min

Temperature  $\vartheta_1$  in the cylinder head: 20.0 °C.

Table 1: Heating voltage  $U$ , heating current  $I$  and temperature increase  $\Delta\vartheta$  of the cooling water as functions of the rotational speed  $f$ .

$\frac{f}{s^{-1}}$	$\frac{U}{V}$	$\frac{I}{A}$	$\frac{\Delta\vartheta}{^\circ C}$
2.7	7.5	1.7	2.4
2.9	7.6	1.8	2.5
3.2	8.0	1.9	2.7
3.4	8.2	2.0	2.9
3.7	8.6	2.1	3.2
4.0	9.0	2.2	3.5

### Evaluation and results

Table 2: Heat  $Q_1$  transferred to the cooling water  $Q_1$ , frictional work of the piston  $W_R$ , heat  $Q_2$  extracted from the cylinder head, and mechanical work  $W$  supplied to the thermodynamic cycle as functions of the rotational speed  $f$  (all quantities per revolution).

$\frac{f}{s^{-1}}$	$\frac{Q_1}{J}$	$\frac{W_R}{J}$	$\frac{Q_2}{J}$	$\frac{W}{J}$
2.7	9.7	1.1	4.7	3.9
2.9	9.4	1.1	4.7	3.6
3.2	9.2	1.0	4.8	3.4
3.4	9.3	1.0	4.8	3.5
3.7	9.4	1.0	4.9	3.5
4.0	9.5	1.0	5.0	3.5

Table 3: the efficiency  $\eta$  of the refrigerator as a function of the rotational speed  $f$ .

$\frac{f}{s^{-1}}$	$\eta$
2.7	1.2
2.9	1.3
3.2	1.4
3.4	1.4
3.7	1.4
4.0	1.4

In Table 2, the heat  $Q_1$  supplied to the cooling water (calculated according to (V)), the frictional work of the piston  $W_R$  (taken from experiment P2.6.1.1), the heat  $Q_2$  extracted from the cylinder head (calculated according to (III)), and the mechanical work  $W$  supplied to the thermodynamic cycle (calculated according to (VI)), all during one revolution, are listed. Fig. 3 is a plot of  $Q_2$  and  $W$  as functions of the rotational speed  $f$ .

The efficiency  $\eta$  is calculated from  $Q_2$  and  $W$  according to (II) and  $Q_2$ . Within the accuracy of measurement, it is constant for rotational speeds above  $f = 3,0 s^{-1}$  (see Table 3), but it is far below the values achieved by refrigerators used in practice.

Fig. 3 The heat (●) and the mechanical work  $W$  (■) as functions of the rotational speed  $f$ .

